ONLINE APPENDIX TO

"OFFSETTING DISAGREEMENT"

Table A1. Belief Crossing and CEF Discounts: Controlling for Investor Sentiment

This table replicates Table 2, but now controls for investor sentiment (while omitting year-quarter fixed effects). In Column 1, *Sentiment* is the Consumer Confidence Index as compiled by The Conference Board; in Column 2, we use the Baker and Wurgler sentiment index; in Column 3, we use the Consumer Sentiment Index computed by the University of Michigan. Our controls are identical to those in Table 2. All independent variables are normalized to have a standard deviation of one. We include fund-fixed effects. *T*-statistics are reported in parentheses and are based on standard errors clustered by fund and year-quarter. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	Consumer Confidence Index by The Conference Board (1)	Baker-Wurgler Sentiment Index (2)	Consumer Sentiment Index by the University of Michigan (3)
InvCov	-0.520***	-0.561***	-0.577**
	(-2.61)	(-2.78)	(-2.42)
Disagreement	0.252	0.181	0.282
	(0.60)	(0.55)	(0.81)
Crossing	0.106	0.175	0.153
	(0.64)	(0.98)	(0.88)
Sentiment	0.125	-0.085	-0.0679
	(0.21)	(-0.87)	(-1.15)
*Sentiment * ln(MarketCap)*	-0.402	0.020	0.022
	(-1.05)	(1.31)	(0.55)
*Sentiment * IO*	-0.508	-0.637	0.061
	(-1.14)	(-0.13)	(1.60)
*Sentiment * Idiosyncratic Volatility*	-0.029	0.0211	0.0702
	(-0.12)	(1.13)	(0.25)
ln(MarketCap)	-2.502	-1.545	-4.058
	(-1.13)	(-1.05)	(-0.79)
IO	0.192	-0.645	-0.058*
	(0.62)	(-1.56)	(-1.73)
Idiosyncratic Volatility	-0.550	0.196	-0.946
	(-0.95)	(0.19)	(-0.40)
Controls	Yes	Yes	Yes
#Obs.	1,906	1,906	1,906
Adj. R^2	0.830	0.832	0.830

Table A2. Belief Crossing and Future Returns

This table documents coefficient estimates from two sets of pooled OLS regression of future one-year return on a measure of investor disagreement and belief crossing. In Column 1, we report coefficient estimates from pooled OLS regressions of one-year four-factor adjusted returns of CEFs on *InvCov*. The dependent variable is the future CEF's one-year four-factor adjusted return (based on prices, not NAVs). In Column 2, we report coefficient estimates from regressions of post-M&A one-year returns on investor disagreement and belief crossing about the acquirer and the target. The dependent variable is the one-year post-M&A DGTW adjusted stock return. We construct *InvCov* as follows: For each stock pair involving securities of the CEF's top-ten holdings, we compile a list of brokerage houses that cover both firms and we compute the Spearman rank correlation in earnings forecasts between these two firms; we also compute the forecast dispersion for each of the two firms. *PairwiseCov* is the product of the Spearman rank correlation and the average forecast dispersion. In Column 1, we aggregate *PairwiseCov* to *InvCov* as the portfolio-weighted average *PairwiseCov* across all stock pairs, multiplied by negative one. In Column 2, *InvCov* is simply *PairwiseCov*, multiplied by negative one. A large positive realization of *InvCov* suggests a high level of belief crossing. In Column 1, our controls are identical to those in the CEF discount regression. In Column 2, our controls are identical to those in the Combined-Announcement-Day-Return regression. All independent variables are normalized to have a standard deviation of one. *T*-statistics are reported in parentheses and are based on standard errors clustered by time. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	Future CEF Returns (1)	Post-M&A Return (2)
InvCov	0.0044*	0.034**
	(1.81)	(2.37)
Disagreement	-0.001**	0.028*
	(0.16)	(1.94)
Crossing	-0.002	0.003
	(-0.83)	(0.20)
Controls	Yes	Yes
#Obs.	6,216	392
Adj. R^2	0.319	0.180

Table A3. Belief Crossing and Exchange-Traded Fund Flows

This table reports coefficient estimates from pooled OLS regressions of monthly ETF flows on a measure of investor disagreement and belief crossing. The dependent variable is the percentage change in the number of shares outstanding of the ETF. The independent variables are as in Table 5, but now represent quarterly changes (rather than levels). All independent variables are normalized to have a standard deviation of one. We include year-quarter-fixed effects (we no longer include fund-fixed effects since all of our variables are now first-differenced). *T*-statistics are reported in parentheses and are based on standard errors clustered by both fund and year-quarter. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	(1)
Δ*InvCov*	-0.380***
	(-3.05)
Δ*Disagreement*	-0.434**
	(-2.48)
Δ*Crossing*	0.165
	(1.40)
Δ*Dividend Yield*	0.437*
	(1.75)
Δ*Liquidity Ratio*	-1.277***
	(-8.54)
Δ*Expense Ratio*	-0.024
	(-0.30)
Δ*Excess Idiosyncratic Volatility*	-0.517***
	(-4.71)
Δ*Excess Skewness*	0.057
	(0.32)
Lagged Returns	Yes
Lagged Flows	Yes
# Obs.	8,092
Adj. R^2	0.026

Context for "Table A4. ETF Spillover Effects"

The results in our paper indicate that authorized participants are price stabilizing. However, there is a flip-side to this arbitrage mechanism. Consider an ETF holding three securities, A, B and C. Assume that investors strongly disagree about the values of A and B and that disagreement offsets when the two securities are viewed as a whole. Investors, in the meanwhile, disagree little about security C.

In the absence of arbitrage forces, the ETF will trade at a discount relative to its underlying assets because of its holdings in A and B. If authorized participants redeem ETF shares and sell the underlying portfolio and/or if other smart investors buy ETF shares and simultaneously short the underlying portfolio to take advantage of the ETF discount, the price of C may fall initially due to the selling pressure, only to rebound subsequently. Note that the spillover mechanism we describe here is *independent* from the investor disagreement channel tested in prior literature, as there is little investor disagreement regarding stock C.

To explore this idea, we construct a measure of *PeerInvCov*. For all non-top-ten stocks, *PeerInvCov* equals *InvCov* computed across the top-ten holdings. For top-ten stocks, *PeerInvCov* is the *InvCov* computed across the other nine top-ten stocks. For each stock, we take the TNA-weighted average *PeerInvCov* across all ETF holdings in that stock. If arbitrage trades – both from authorized participants and other smart investors – have a meaningful impact on the price of C (due to price pressure), we expect *PeerInvCov* to negatively associate with contemporaneous stock returns, but to positively predict future returns.

We start with checking the contemporaneous return patterns and we employ the same event window as in the flow test.[1] Specifically, we examine how stock returns in the three months (i.e., -1, 0, 1) surrounding the ETF's quarter end reporting date relate to quarterly *changes* in *PeerInvCov*. We focus on the change rather than level of *PeerInvCov* because it is the shock to *PeerInvCov* (a highly persistent variable) that triggers arbitrage trading (similar to Δ*InvCov* triggering ETF flows). Further, since the return effect should be concentrated in the part of the sample with large Δ*PeerInvCov*, we compare stocks in the top quintile in terms of Δ*PeerInvCov* with stocks in the other four quintiles. (Our results would go through if we instead compare the top Δ*PeerInvCov* quintile with the bottom quintile.) Consistent with our conjecture, in untabulated analyses, we find that the average cumulative four-factor alpha of the top quintile is -1.2% (*t*-statistics = -1.77) while the average alpha of stocks in the other four quintiles is 0.12% (*t*-statistics = 1.10). The difference between the two is statistically significant (Δ = -1.56%, *t*-statistics = -2.31).

We next examine whether stock prices bounce back after the initial drop. To this end, we employ both a portfolio approach and a Fama-MacBeth (1973) regression analysis. In the calendar-time portfolio test, we sort stocks into quintiles based on *PeerInvCov* as of month zero, and go long the top quintile and short the bottom four quintiles from months 2 through 6. We argue that for the purpose of detecting the return reversal, the level of *PeerInvCov* reflects the cumulative effect of arbitrage trades to date and is thus the right variable to focus on. As shown in Panel A of Table A4, stocks in the top quintile outperform their peers, on a four-factor adjusted basis, by 0.34% (*t*-statistic = 2.27) to 0.50% (*t*-statistic = 3.07) per month in these five months, or by 1.7% to 2.45% over the entire period. The magnitude of the reversal pattern lines up well with the magnitude of the initial price drop.

In Panel B, we estimate Fama-MacBeth (1973) regressions. The dependent variable is the monthly DGTW-adjusted return. The independent variable of primary interest is each stock's *PeerInvCov*. We also include in the regression the stock's own earnings forecast dispersion, and other controls that are known to forecast future stock returns. All independent variables are normalized to have a standard deviation of one with the exception of *PeerInvCovQR*, which is the quintile ranking of *PeerInvCov*.

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[1] Our sample consists of NYSE/AMEX/NASDAQ common stocks with price-per-share greater than $5 and with fraction of shares held by ETFs greater than the median of its distribution.

As shown in Column 2 of Panel B, after controlling for the stock's own earnings forecast dispersion, the coefficient estimate on the quintile dummy *PeerInvCovQR* is 0.102 (*t*-statistic = 2.78). This implies that stocks in the top quintile outperform those in the bottom quintile by nearly 41bps (0.102 * 4 = 0.408) per month in the next five months. The point estimate increases to 0.150 (*t*-statistic = 2.43) if in each cross section, we estimate a weighted-least-square regression where the weight is proportional to each stock's lagged market capitalization. Overall, the evidence confirms our prediction that arbitrage trades that are aimed to correct the discrepancy between the ETF value and underlying portfolio value can sometimes have a destabilizing effect on some of the underlying securities.

Table A4. ETF Spillover Effects

This table reports the profitability of a trading strategy exploiting the ETF spillover effect. For each stock, we construct *PeerInvCov* as discussed above. The sample consists of common stocks traded in NYSE/AMEX/NASDAQ for which the price-per-share is greater than $5 and fraction of shares held by ETFs is greater than the median of its distribution. We skip a month after portfolio formation and we hold the portfolios for six months. Panel A reports the monthly Carhart (1997) factor alphas for the "highest *PeerInvCov* quintile" portfolio and the "remaining four quintile" portfolio. Panel B reports coefficient estimates from monthly Fama-MacBeth (1973) regressions of DGTW-adjusted returns [%] on *PeerInvCov*. We use quintile rankings (0-4) for *PeerInvCov*; all other independent variables are normalized to have a standard deviation of one. In Columns 1 and 2 of Panel B, we equal-weight each month. In Columns 3 and 4 of Panel B, we weight by market capitalization. *T*-statistics are based on Newey-West (1987) standard errors with six lags and are reported in parentheses. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

Panel A: Portfolio Approach

	Equal-Weighted	*T*-Statistics	Value-Weighted	*T*-Statistics
Top *PeerInvCov* Quintile	0.35%	[2.55]	0.42%	[2.93]
Other *PeerInvCov* Quintiles	0.01%	[0.14]	-0.08%	[-2.02]
Top-Other	0.34%	[2.27]	0.50%	[3.07]

Panel B: Regression Approach

	OLS (1)	OLS (2)	WLS (3)	WLS (4)
PeerInvCov	0.119***	0.102***	0.187**	0.150**
	(2.70)	(2.78)	(2.13)	(2.43)
Ln(MarketCap)		-0.148**		-0.103
		(-2.55)		(-0.54)
Book-to-Market Ratio		-0.050		0.042
		(-0.80)		(0.28)
Past-One-Year Returns		-0.067		0.030
		(-0.36)		(0.11)
Turnover		-0.167		0.713***
		(-1.37)		(2.83)
Dispersion		-0.215***		-0.984***
		(-2.99)		(-2.68)
Volatility		-0.012		-0.729***
		(-0.18)		(-2.63)
# Qtrs.	36	36	36	36
Adj. R^2	0.003	0.038	0.022	0.143

Table A5. Belief Crossing and Operating Performance of the Combined Firm

This table reports coefficient estimates from regressions of post-M&A operating performance measures on a measure of investor disagreement and belief crossing about the acquirer and the target. The dependent variable is the post-M&A five-year average of *ROA*, *ROE*, *Profitability* and *Sales Growth*. We construct *InvCov* as follows: We compile a list of brokerage houses that cover both the acquirer and the target and we compute the Spearman rank correlation in earnings forecasts between these two firms; we also compute the forecast dispersion for each of the two firms. *InvCov* is the product of the Spearman rank correlation and the average forecast dispersion, multiplied by negative one. A large positive realization of *InvCov* suggests a high level of embedded belief crossing. In Panels B and C, we augment *InvCov* with (1-*IO*) and with (1-*IO*) * *SI*, respectively, where *IO* is the residual institutional ownership and *SI* is short interest. Our controls are identical to those in the Combined-Announcement-Day-Return regression. All independent variables are normalized to have a standard deviation of one. We include year-fixed effects. *T*-statistics are reported in parentheses and are based on standard errors clustered by year. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	ROA	*ROE*	*Profitability*	*Sales Growth*
	(1)	(2)	(3)	(4)
InvCov	0.000	0.003	0.008	0.003
	(0.00)	(0.29)	(1.01)	(0.38)
Disagreement	-0.005*	-0.013*	-0.006	-0.009
	(-1.96)	(-1.79)	(-0.97)	(-1.43)
Crossing	-0.001	0.001	0.002	0.005
	(-0.40)	(0.09)	(0.29)	(0.71)
#Obs.	363	363	363	363
Adj. R^2	0.568	0.451	0.349	0.281

Table A6. Belief Crossing and Likelihoods of CEF/ETF IPOs

This table reports coefficient estimates from a logit regression of sector-CEF/ETF IPOs on a measure of investor disagreement and belief crossing about the sector. Specifically, we compute, for each two-digit SIC-code industry in each year-quarter, the average level of embedded belief crossing across all stock pairs within that industry. We then examine whether the creation of CEFs and ETFs specializing in that industry is tied to the corresponding level of belief crossing. The dependent variable equals one if the industry/year-quarter has at least one CEF or ETF IPO specializing in that industry, and zero otherwise. The independent variables include belief crossing (*InvCov*), market capitalization, book-to-market ratio and past one-year returns, all at the industry/year-quarter level. All independent variables are normalized to have a standard deviation of one. Z-values are reported in parentheses and are based on standard errors clustered by year-quarter. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	Sector Fund IPOs (1)
InvCov	-0.146*** (-2.80)
Disagreement	0.145 (1.11)
Crossing	0.191 (1.14)
Industry Characteristics:	Yes
# Obs.	816
Pseudo R2	0.049

Table A7. Belief Crossing and Likelihoods of Mergers and Acquisitions

This table reports coefficient estimates from logit regressions of M&A announcements on a measure of investor disagreement and belief crossing about actual acquirer-target pairs and pseudo acquirer-target pairs. For each M&A announcement in our sample, we construct a set of counterfactual firm pairs, which are similar to the actual M&A pair along an array of observable firm characteristics, but involve firms that did not engage in an M&A. Specifically, for each firm involved in an M&A, we identify ten pseudo acquirers and ten pseudo targets that are in the same two-digit-SIC-code industry as, and are the closest to, the actual acquirer and the actual target along the dimensions of firm size, book-to-market ratio and past one year returns, using a propensity score matching approach. In Column (1), we individually match the actual acquirer with each of the ten pseudo targets, resulting in ten counterfactual firm pairs. In Column (2), we reverse the matching and individually match the actual target with each of the ten pseudo acquirers, resulting in ten counterfactual firm pairs. In Column (3), we match each of the ten pseudo acquirers with each of the ten pseudo targets, resulting, again, in ten counterfactual firm pairs. The dependent variable equals one for actual M&A pairs, and zero for counterfactual firm pairs. *InvCov* is the level of belief crossing of the actual- and the pseudo acquirer-target pairs. The remaining independent variables are the same as in the combined-announcement day-return regression, but are now averaged to the firm-pair level. All independent variables are normalized to have a standard deviation of one. *Z*-values are reported in parentheses and are based on standard errors clustered by year. Statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

	Pseudo Target Only (1)	Pseudo Acquirer Only (2)	Pseudo Acquirer and Pseudo Target (3)
InvCov	-0.090***	-0.134***	-0.132***
	(-2.73)	(-3.94)	(-4.13)
Disagreement	0.069	-0.081	-0.084
	(0.91)	(-1.11)	(-1.11)
Crossing	0.261**	0.419***	0.323***
	(2.46)	(3.92)	(3.20)
Acquirer Characteristics:	Yes	Yes	Yes
Target Characteristics:	Yes	Yes	Yes
Deal Characteristics:	Yes	Yes	Yes
# Obs.	3,091	3,630	3,740
Pseudo R2	0.038	0.019	0.023

Figure A1. Survey Design

Q1: Have you ever invested in either

- a **mutual fund**,
- an **exchange-traded fund,**
- or a **closed-end fund**,

and the fund was NOT just mimicking a broader index such as the S&P 500 (i.e., the fund was NOT an index fund)?

○ Yes
○ No

The following questions will ask you which sources of information you used when investing in a fund.

The sources, along with hyperlinks of what some of these sources likely looked like, are as follows:

- **Sources**:
    - The fund's website
    - Some other investment related website
    - The fund's Fact Sheet (basic 3-4 page document that provides an overview of a fund)
    - The fund's Prospectus (compared with the Fact Sheet, a much longer document that provides details on a fund; primarily filed to inform potential new investors)
    - The fund's Annual Report (compared with the Fact Sheet, a much longer document that provides details on a fund; compared with the Prospectus, more of an ongoing annual "report card" filed to inform new and current investors)
    - Discussion of the fund in a print or online article
    - Recommendation by a family member, friend, colleague, or other acquaintance
    - Recommendation by your financial advisor
    - Advertisement
    - Other

Please click on all the hyperlinks before answering any of the questions below.

Q2: If you ever invested in a fund and the fund was NOT an index fund, which **sources** of information did you use when **first deciding which fund to invest in**? **[click all that apply]**

Sources

- ☐ The fund's website
- ☐ Some other investment related website
- ☐ The fund's Fact Sheet (basic 3-4 page document that provides an overview of a fund)
- ☐ The fund's Prospectus (compared with the Fact Sheet, a much longer document that provides details on a fund; primarily filed to inform potential new investors)
- ☐ The fund's Annual Report (compared with the Fact Sheet, a much longer document that provides details on a fund; compared with the Prospectus, more of an ongoing annual "report card" filed to inform new and current investors)
- ☐ Discussion of the fund in a print or online article
- ☐ Recommendation by a family member, friend, colleague, or other acquaintance
- ☐ Recommendation by your financial advisor
- ☐ Advertisement
- ☐ Other

Q3: **Once invested** in a fund, which **sources** of information did you use when **monitoring your investment** and deciding whether to remain invested in the fund, pull your money out of the fund, or add more money to the fund? **[click all that apply]**

Sources

- ☐ The fund's website
- ☐ Some other investment related website
- ☐ The fund's Fact Sheet (basic 3-4 page document that provides an overview of a fund)
- ☐ The fund's Prospectus (compared with the Fact Sheet, a much longer document that provides details on a fund; primarily filed to inform potential new investors)
- ☐ The fund's Annual Report (compared with the Fact Sheet, a much longer document that provides details on a fund; compared with the Prospectus, more of an ongoing annual "report card" filed to inform new and current investors)
- ☐ Discussion of the fund in a print or online article
- ☐ Recommendation by a family member, friend, colleague, or other acquaintance
- ☐ Recommendation by your financial advisor
- ☐ Advertisement
- ☐ Other

Figure A2. Survey Procedure and Results

We design a Qualtrics survey as shown in Figure A1. We recruit survey participants via Prolific (https://prolific.ac). We require that participants reside in the U.S. and report "yes" to the following two questions within Prolific: (1) "Have you ever made investments (either personal or through your employment) in the common stock or shares of a company?", (2) "Have you invested in any of the following types of investment in the past?: ETF or ETC, Government Bonds or Stock Market." Within our Qualtrics survey, we also ask "Have you ever invested in either a mutual fund, an exchange-traded fund, or a closed-end fund, and the fund was NOT just mimicking a broader index such as the S&P 500 (i.e., the fund was NOT an index fund)," and we require survey participants to respond with "yes." Each participant is paid the equivalent of $30/hour for successful survey completion, which is above the minimum pay requested by Prolific of $7.50/hour. A total of 114 participants completed our survey. We report the survey responses below.

	Sources	Fraction of participants reporting to draw from a particular sources	
		Q2	Q3
(1)	The fund's website	57%	42%
(2)	Some other investment related website	49%	48%
(3)	Fact Sheet	72%	61%
	[Either (1), (2) or (3)]	[93%]	[100%]
(4)	Prospectus	52%	52%
(5)	Annual Report	38%	44%
(6)	Discussion of the fund in a print or online article	33%	28%
(7)	Recommendation by a family member, friend, colleague, or other acquaintance	45%	25%
(8)	Recommendation by your financial advisor	31%	29%
(9)	Advertisement	5%	0%
(10)	Other	75%	61%

Online Appendix
A Model of Investor Disagreement and Belief Crossing

1 Model Setup

The economy lasts for two periods, $t = 0$ and 1. On day 0, a financial market operates and investors trade financial assets. On day 1, asset payoffs are realized. Two assets are traded in the financial market. Without loss of generality, asset $i \in \{1,2\}$ has a positive constant supply, which is normalized to be one unit, and pays a liquidating dividend of $\tilde{f}_i$.

For tractability, all investors are assumed to be risk-neutral. There are four types of investors $j \in \{OO, OP, PO, PP\}$, with different beliefs about assets' final payoffs. "*O*" indicates optimism and "*P*" indicates pessimism. More specifically, type-*OO* investors are optimistic about both assets; type-*PP* investors are pessimistic about both assets; type-*OP* investors are optimistic about asset 1, but pessimistic about asset 2; finally, type-*PO* investors are pessimistic about asset 1, but optimistic about asset 2. We assume that the population of investors that are optimistic (or pessimistic) about both assets (type-*OO* or type-*PP*) is $1-\lambda$, and the population of investors with opposing views (type-*OP* or type-*PO*) is λ ($0 \leq \lambda \leq 1$). In other words, the population of investors that are optimistic (pessimistic) about either asset is exactly 1. Meanwhile, λ captures the degree of "belief crossing" among investors.

We further assume that investors that are optimistic about asset i believe that asset i's final payoff is $1+\sigma$; conversely, investors that are pessimistic about asset i believe that asset i's final payoff is $1-\sigma$. We use $E(\cdot)$ to denote investors' beliefs in the following analysis. Here, σ captures the degree of disagreement among investors. Given limited liability of stocks (i.e., stock payoffs can not be negative), we further assume that $0 < \sigma \leq 1$.

Investors trade both assets to maximize expected utilities based on their beliefs. Trading is costly in the economy. We follow the literature (e.g., Gârleanu and Pedersen, 2013; Dávila and Parlatore, 2018; Huang, Qiu and Yang, 2018) to introduce a quadratic cost function for type-j investors' demand for asset i:

$$\frac{1}{2} \times X_{j,i}^2.$$

This quadratic form of transaction costs is common in the theoretical literature and is a reduced-form approach to modeling transaction frictions. In economic terms, the transaction cost in our setting can be interpreted as commission fees, inventory costs, or operation costs. Consequently, type-j investors' utility optimization problem can be summarized as:

$$\max_{X_{j,1},X_{j,2}} \sum_{i=1}^{2} \left[X_{j,i} E_j(\tilde{f}_i) - X_{j,i}\tilde{p}_i - \frac{1}{2} \times X_{j,i}^2 \right].$$

Without Short-Sale Constraints, it is clear that type-j's demand for asset i (from the first-order condition) is:

$$X_{j,i} = E_j(\tilde{f}_i) - \tilde{p}_i.$$

With Short Sales Constraints, we assume that investors, who want to sell short, can only do so up to a non-negative fraction, β (< 1), of their initial demand. Intuitively, β captures the shorting cost or shorting difficulty. With a larger β, investors face a less binding short-sale constraint. Because the second-order condition from the above optimization is always negative, when $E_j(\tilde{f}_i) - \tilde{p}_i$ is negative, type-j's demand for asset i is: $X_{j,i} = \beta[E_j(\tilde{f}_i) - \tilde{p}_i]$.

Equilibrium: On day 0, the financial market opens and investors submit their demand schedules subject to short-sale constraints. Equilibrium prices are determined by the market clearing condition.

Discussion: In Reed, Saffi and Van Wesep (2016), there are only two types of investors with opposing beliefs about the two assets. In other words, the setting considered by Reed, Saffi and Van Wesep (2016) is a special case of our model with $\lambda = 1$. Consequently, we use our model to highlight the role of both investor disagreement (σ) and belief crossing (λ) in driving portfolio discounts.

2 Benchmark Case: No Short-Sale Constraints

This section solves the benchmark model without short-sale constraints, i.e., when $\beta = 1$.

Individual Assets: Demand by optimistic investors for asset i is $1+\sigma-\tilde{p}_i$, and that by pessimistic investors is $1-\sigma-\tilde{p}_i$. This yields the following set of equilibrium prices:

LEMMA **1.** $\tilde{p}_1 = \tilde{p}_2 = \frac{1}{2}$.

Proof. See the model solution section. □

An Equal-Weight Portfolio: Now consider a portfolio C that has equal weights in both assets. The sum of its components is $\tilde{f}_C = 0.5\tilde{f}_1 + 0.5\tilde{f}_2$. Denote the price of this portfolio $\tilde{p}_C$ (assume it is separately traded). Demand for this portfolio can be characterized as:

$$\begin{aligned} X_{OO,C} &= 1+\sigma-\tilde{p}_C, \\ X_{OP,C} &= 1-\tilde{p}_C, \\ X_{PO,C} &= 1-\tilde{p}_C, \\ X_{PP,C} &= 1-\sigma-\tilde{p}_C. \end{aligned}$$

Market clearing then implies:

LEMMA **2.** $\tilde{p}_C = \frac{1}{2}$.

Proof. See the model solution section. □

It is clear from Lemma 1 and Lemma 2 that absent short-sale constraints, the portfolio value is equal to the sum of its parts.

3 General Case: with Short-Sale Constraints

This section solves the model with short-sale constraints, i.e., when $0 \leq \beta < 1$.

Individual Assets: In the presence of short-sale constraints, pessimistic investors, who want to sell short, are unable to sell to the full extent. Consequently, asset prices do not fully reflect pessimists' views and are biased upward. Moreover, holding the average investor belief constant, when investors disagree more strongly, equilibrium asset prices are more upward biased:

LEMMA **3.** *When* $\sigma \leq \frac{1}{2}$*, the prices of both assets are* $\tilde{p}_1 = \tilde{p}_2 = \frac{1}{2}$*; When* $\sigma > \frac{1}{2}$*, the prices of both assets are* $\tilde{p}_1 = \tilde{p}_2 = \frac{\beta+\sigma(1-\beta)}{1+\beta} > \frac{1}{2}$.

Proof. See the model solution section. □

An Equal-Weight Portfolio: We again consider a portfolio that invests 50-50 in both assets. Market clearing implies:

LEMMA **4.** *When* $\sigma \leq \frac{1}{2}$*, the price of the portfolio is:* $\tilde{p}_C = \frac{1}{2}$*; When* $\sigma > \frac{1}{2}$*, the price of the portfolio is:* $\tilde{p}_C = \frac{\lambda+(1-\lambda)\beta+\sigma(1-\lambda)(1-\beta)}{1+\lambda+(1-\lambda)\beta}$.

Proof. See the model solution section. □

We define the portfolio discount as the difference between the portfolio value and the sum of its components' value: $discount = 1 - \tilde{p}_C/(0.5\tilde{p}_1 + 0.5 * \tilde{p}_2)$.

PROPOSITION **1.** *When* $\sigma \leq \frac{1}{2}$*, the portfolio value is equal to the sum of its components. That is:* $\tilde{p}_C = 0.5\tilde{p}_1 + 0.5\tilde{p}_2$*. When* $\sigma > \frac{1}{2}$*, the portfolio value is less than the sum of its components. That is:* $\tilde{p}_C < 0.5\tilde{p}_1 + 0.5\tilde{p}_2$.

Proof. See the model solution section. □

Proposition 1 shows that the presence of short-sale constraints is a necessary condition to generate a portfolio discount. Lemma 4 further shows that, with short-sale constraints, both disagreement σ and the degree of "belief crossing" λ affect the portfolio value and consequently the discount. Formally, the joint effect of disagreement and "belief crossing" on portfolio discounts satisfies the following condition:

PROPOSITION **2.** *When* $\sigma \leq \frac{1}{2}$, $\frac{\partial^2 discount}{\partial\sigma\partial\lambda} = 0$*; When* $\sigma > \frac{1}{2}$, $\frac{\partial^2 discount}{\partial\sigma\partial\lambda} > 0$;

Proof. See the model solution section. □

4 Model Solution

This section provides all proofs omitted above with auxiliary results.

PROOF OF LEMMA 1. When there are no short-sale constraints, all investors trade the asset i ($i = 1$ and 2). Inserting investors' demands into the market clearing condition, we get

$$1 + \sigma - \tilde{p}_i + 1 - \sigma - \tilde{p}_i = 1, \tag{4.1}$$

and then directly solve the price of asset i as $\tilde{p}_i = \frac{1}{2}$.

□

PROOF OF LEMMA 2 . When there are no short-sale constraints, all investors trade the portfolio. Inserting investors' demands into the market clearing condition, we get

$$(1-\lambda)X_{OO,C} + \lambda X_{OP,C} + (1-\lambda)X_{PO,C} + \lambda X_{PP,C} = 2 - 2\tilde{p}_C = 1. \tag{4.2}$$

We directly solve the price of the portfolio as $\tilde{p}_C = \frac{1}{2}$.

□

PROOF OF LEMMA 3 . With short-sale constraints, the equilibrium prices depend on whether the pessimistic investors long or short the assets. With the symmetry, we can focus on one asset, i.e., asset 1. There are two scenarios: the pessimistic investors (type$-PO$ and type$-PP$) long the asset 1, and the pessimistic investors short the asset 1. We solve the price of asset 1 for these two scenarios as follows.

1. If pessimistic investors long the asset 1, inserting investors' demands into the market clearing condition, we get

$$1 + \sigma - \tilde{p}_i + 1 - \sigma - \tilde{p}_i = 1, \tag{4.3}$$

and then solve the price of asset 1 as $\tilde{p}_i = \frac{1}{2}$. After that, we need to ensure that the pessimistic investors indeed long the asset 1 in the equilibrium. When $\tilde{p}_i = \frac{1}{2}$, the demand of pessimistic investors is $\frac{1}{2} - \sigma$. It is clear that when $\sigma \leq \frac{1}{2}$, pessimistic investors long the asset 1.

2. If the pessimistic investors short the asset 1, inserting investors' demands into the market clearing condition, we get

$$1 + \sigma - \tilde{p}_i + \beta(1 - \sigma - \tilde{p}_i) = 1, \tag{4.4}$$

in which only a proportion of shorted shares (β) can be fulfilled. From the market clearing condition, we solve the price of asset 1 as

$$\tilde{p}_i = \frac{\beta + \sigma(1-\beta)}{1+\beta}. \tag{4.5}$$

After that, we need to ensure that the pessimistic investors indeed short the asset 1 in the equilibrium. When $\tilde{p}_i = \frac{\beta+\sigma(1-\beta)}{1+\beta}$, the demand of pessimistic investors is $1 - \sigma - \frac{\beta+\sigma(1-\beta)}{1+\beta}$, which equals $\frac{1-2\sigma}{1+\beta}$. It is clear that when $\sigma > \frac{1}{2}$, pessimistic investors short the asset 1.

□

PROOF OF LEMMA 4 . Different from the Lemma 3, there are two groups of pessimistic investors: one group consists of type$-OP$ and type$-PO$, and the other group only consists of type$-PP$. It is clear that the first group has larger demand for the portfolio than the second group. There are three potential scenarios, which depend on the shorting status of different groups of pessimistic investors. We solve the price of the portfolio in different scenarios as follows.

1. If both groups of pessimistic investors long the portfolio, inserting investors' demands into the market clearing condition, we get

$$(1-\lambda)X_{OO,C} + \lambda X_{OP,C} + (1-\lambda)X_{PO,C} + \lambda X_{PP,C} = 2 - 2\tilde{p}_C = 1. \tag{4.6}$$

Then we solve the price of the portfolio as $\tilde{p}_C = \frac{1}{2}$. After that, we need to ensure that all pessimistic investors indeed long the portfolio in the equilibrium. To find the condition for all investors to long the portfolio, we can only focus on the demand of the most pessimistic investors (type$-PP$). The demand of type$-PP$ is $2 - 2\sigma - 2\tilde{p}_C$, which equals $1 - 2\sigma$. It is clear that when $\sigma \leq \frac{1}{2}$, all pessimistic investors long the portfolio.

2. If the first group of pessimistic long the portfolio and the second group short the portfolio, inserting investors' demands into the market clearing condition, we get

$$\begin{aligned} & (1-\lambda)X_{OO,C}+\lambda X_{OP,C}+(1-\lambda)\beta X_{PP,C}+\lambda X_{PO,C} && (4.7)\\ = \; & (1-\lambda)(1+\sigma-\tilde{p}_C)+2\lambda(1-\tilde{p}_C)+(1-\lambda)\beta(1-\sigma-\tilde{p}_C) && (4.8)\\ = \; & 1. && (4.9) \end{aligned}$$

(Note: only a proportion of shorted shares (β) from the second group can be fulfilled). We solve the price of the portfolio as

$$\tilde{p}_C=\frac{\lambda+(1-\lambda)\beta+\sigma(1-\lambda)(1-\beta)}{1+\lambda+(1-\lambda)\beta}. \quad (4.10)$$

After that, we need to ensure that the most pessimistic investors indeed short the portfolio and the least pessimistic investors indeed long the portfolio. The demand of the most pessimistic investors (type$-PP$) is negative, which yields

$$X_{PP,C}<0 \quad (4.11)$$

$$\Leftrightarrow \sigma>\frac{1}{2}. \quad (4.12)$$

The demand of the least pessimistic investors (type$-PO$ or type$-OP$) is non-negative, which yields

$$X_{PO,C}\geq 0 \quad (4.13)$$

$$\Leftrightarrow \sigma\leq\frac{1}{(1-\lambda)(1-\beta)}. \quad (4.14)$$

Because $\frac{1}{(1-\lambda)(1-\beta)}>1$, so σ is always smaller than $\frac{1}{(1-\lambda)(1-\beta)}$ under the assumption that $\sigma\leq 1$.

3. If both groups of pessimistic investors short the portfolio, inserting investors' demands into

the market clearing condition, we get

$$
\begin{aligned}
& (1-\lambda)X_{OO,C}+\lambda\beta X_{OP,C}+(1-\lambda)\beta X_{PP,C}+\lambda\beta X_{PO,C} && (4.15)\\
= \;& (1-\lambda)(1+\sigma-\tilde{p}_C)+2\lambda\beta(1-\tilde{p}_C)+(1-\lambda)\beta(1-\sigma-\tilde{p}_C) && (4.16)\\
= \;& 1. && (4.17)
\end{aligned}
$$

We solve the portfolio price as:

$$\tilde{p}_C=\frac{\sigma-\lambda(1+\sigma)+2\lambda\beta+(1-\lambda)\beta(1-\sigma)}{(1-\lambda)+\beta+\lambda\beta}. \tag{4.18}$$

After that, we need to ensure that both groups of pessimistic investors indeed short the portfolio in the equilibrium. Given that the less pessimistic investors have higher demand than the most pessimistic ones, we can focus on the demand of least pessimistic investors. The demand of the least pessimistic investors (i.e., type$-OP$) is negative, which yields

$$\frac{\sigma-\lambda(1+\sigma)+2\lambda\beta+(1-\lambda)\beta(1-\sigma)}{(1-\lambda)+\beta+\lambda\beta}>1 \tag{4.19}$$

$$\Leftrightarrow \sigma>\frac{1}{(1-\lambda)(1-\beta)}. \tag{4.20}$$

Under the assumption that $\sigma\leq 1$, the above inequality is impossible. Thus, only Scenario 1 and 2 are relevant. □

PROOF OF PROPOSITION 1. We prove this proposition for different levels of σ: $\sigma\leq\frac{1}{2}$ and $\sigma>\frac{1}{2}$.
(1) When $\sigma\leq\frac{1}{2}$, the portfolio price is $1/2$, which equals the equal-weighted underlying asset prices.
(2) When $\sigma>\frac{1}{2}$, the difference between the portfolio price and the equal-weighted underlying asset prices is

$$\frac{\lambda+(1-\lambda)\beta+\sigma(1-\lambda)(1-\beta)}{1+\lambda+(1-\lambda)\beta}-\frac{\beta+\sigma(1-\beta)}{1+\beta}, \tag{4.21}$$

which equals

$$-\frac{\lambda(1-\beta)}{1+\beta}\frac{(2\sigma-1)}{1+\lambda+(1-\lambda)\beta}. \tag{4.22}$$

Because $\sigma > \frac{1}{2}$ and $\beta < 1$, the above equation is always negative. This suggests that the portfolio price is smaller than the equal-weighed underlying asset prices.

□

PROOF OF PROPOSITION 2. We prove this proposition for different levels of σ: $\sigma \leq \frac{1}{2}$ and $\sigma > \frac{1}{2}$.

(1) When $\sigma \leq \frac{1}{2}$, the portfolio price is $1/2$. It is easy to get that $\frac{\partial^2 discount}{\partial\sigma\partial\lambda} = 0$.

(2) When $\sigma > \frac{1}{2}$, the direct calculation of $\frac{\partial^2 discount}{\partial\sigma\partial\lambda}$ shows:

$$\frac{\partial^2 discount}{\partial\sigma\partial\lambda} = \frac{(1-\beta)(\beta+1)^2}{(-\beta\lambda+\beta+\lambda+1)^2(-\beta\sigma+\beta+\sigma)^2} > 0, \tag{4.23}$$

where $\beta < 1$. This completes the proof.

□

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